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By

Robert M. Gagné
Princeton University

and Noel E. Paradise
University of Maryland

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ABILITIES AND LEARNING SETS IN KNOWLEDGE ACQUISITION[1]

ROBERT M. GAGNÉ
Princeton University

AND

NOEL E. PARADISE
University of Maryland

THERE is considerable current interest in the techniques of programed learning, as well as in the experimental investigation of the nature of behavioral processes involved in their use (Lumsdaine & Glaser, 1960). It has been proposed (Gagné, in press) that the learning that transpires in the course of administration of learning programs may be called *productive learning,* and distinguished in operational terms from the *reproductive learning* of the typical verbal learning experiment. Whereas in the latter type, performance is usually measured on the specific task that is practiced during learning, in the former, measures of terminal performance are designed to demonstrate mastery of the defined *class* of tasks. For example, learning programs are intended to establish proficiency in such classes of tasks as "adding binary numbers," or "solving linear equations," and the specific tasks employed to measure such performance are considered merely representative of the total class.

Individual differences, naturally enough, are prominently observable in productive learning situations, perhaps even more than they are in verbal learning studies of the reproductive sort. It is not uncommon, for example, to find that fast learners complete a learning program in half the time of slow learners.[2] This fact has sometimes been cited (Skinner, 1958) as an advantage to the use of programed learning, contrasted to the usual classroom learning, in that individuals are allowed to proceed at their own pace depending upon their initial ability. However, there appears to be little if any current evidence about the nature of these individual differences in completion of learning programs, beyond the fact that they occur.

One hypothesis about the origin of differences in rate of completion of a learning program is that rate of acquisition of the successive items (frames) of the program is basically determined by an ability which may be called "general intelligence." Testing this hypothesis would presumably involve the partialing out (experimentally or otherwise) of other factors which might affect speed of program completion, such as "reading speed" and "speed set." This would by no means be easy to do. Besides this such a hypothesis about general intelligence tends to restore an older, original meaning to the latter phrase, that is, "learning rate" ability. Undoubtedly such a restoration would be welcome to many psychologists, if verification could be obtained for it. However, it would also run contrary to the general trend of results obtained over a period of years, most of which have failed to obtain evidence of a general factor which might be called

[1] This study results in part from a collaborative effort between a project on mathematical concept learning, directed by the senior author at Princeton University, and the University of Maryland Mathematics Project, with which both authors have been associated. Gratitude is expressed to John Mayor and Helen Garstens, the Director and Associate Director of the latter project, for their encouragement and assistance in carrying out this study, and also for their forbearance in countenancing the use of mathematical ideas for which they would hasten to disclaim responsibility. In part also, this study owes its support to funds granted by the Carnegie Corporation of New York. The opinions expressed are those of the authors, and do not necessarily reflect the views of any of these supporting organizations.

[2] This was the finding in an unpublished study which utilized the learning program on equation solving described herein—GAGNÉ, R. M., & DICK, W. Learning measures in a self-instructional program in solving equations.

learning rate ability that is common to achievement in a variety of different learning situations (cf. Woodrow, 1946).

An alternative hypothesis to account for individual differences in rate of completion of, and achievement in, learning programs has been proposed (Gagné, in press). This is to the effect that such observed differences result primarily from the fact that individuals begin the task of learning with different amounts and kinds of *knowledge*. Knowledge relevant to any given final task to be learned is conceived as a set of subordinate capabilities called *learning sets* (cf. Harlow, 1949). These are considered to be arranged in a hierarchy such that any learning set may have one or more learning sets subordinate to it in the sense that they mediate positive transfer to the given learning set. These subordinate sets in turn have other learning sets subordinate to them, and so on.

Each learning set in the hierarchy is represented by a distinct class of tasks, and measured in the individual by one or more representative tasks from this class. In order for learning to occur at any point in the hierarchy, according to this theory, each of the learning sets subordinate to a given task must be highly recallable, and integrated by a thinking process into the solution of the problem posed by the task. The attainment of the final task is thus conceived to be a matter of successive attainment and "integration" of a series of lower level learning sets, *beginning with those which are already available* to the individual. Failure to achieve the final task may have several causes: a subordinate learning set may have been omitted from the program, and therefore could not have been acquired; insufficient practice (or other condition) may have resulted in low recallability of one or more subordinate learning sets; or the program may have been unsuccessful in inducing the integration necessary for the attainment of any given learning set in the hierarchy.

The hierarchy of learning sets which supports any given final task may be defined by an analysis procedure. The question is asked of the task, and successively of each task thereby defined: "What would the individual have to know how to do in order to be able to achieve this (new) task, when given only instructions?" The answer to this question defines one or more subordinate learning sets, and each of these in turn may be seen to be dependent upon one or more subordinate learning sets, until the entire hierarchy is defined (see Figure 1 below). At the bottom of this hierarchy are some learning sets which are very simple and very general indeed. Although they are arrived at in the manner described, they seem to resemble, as a class, those tasks presented by tests used to measure certain very general abilities which have been identified by factor analysis techniques. For example, a learning set hierarchy for the task of adding dissimilar fractions may be found to contain at its lowest level such learning sets as "adding two-digit numbers," "multiplying two-digit by one-digit numbers;" and "recognizing symbols." These appear to be the tasks represented by tests of Number Ability and Symbol Recognition (a variety of Associative Memory) (French, 1954). We shall examine additional implications of this finding in a moment.

In accordance with this conception of the learning set hierarchy, we can begin to explore an expected set of correlates (predictor variables) for the individual differences to be observed in connection with the administration of a learning program. First of all, individuals would be expected to begin a learning program with different numbers and kinds of learning sets. Many of these will of course be *irrelevant* to the task, that is, they will not have been identified as members of the hierarchy derived in the manner previously described. More importantly, however, individuals will differ in the pattern of learning sets they possess within the hierarchy of *relevant* learning sets. Some may possess only a few, all at the lowest level; others may possess nearly all that are required to achieve the final performance. Still others may display rather uneven patterns, indicating "gaps" in their previously acquired knowledge. The number and kind of learning sets that the learner brings to the learning program situation may be expected to determine *how rapidly he completes it, other things being equal.*

For an individual who already has many of the relevant learning sets, responding to the frames of a learning program will be largely a matter of review, of traveling over familiar ground. For a person with only a few low level learning sets to begin with, the attainment of each new one may be expected to take time greater than is needed for "review"; and more of these will need to be acquired in order to attain successful performance on the final task.

A second source of individual differences has its origin in the fact that different individuals approach the learning with different patterns of "basic abilities," which may turn out to be those factors which appear consistently and dependably in factor studies (French, 1954). Some of these basic abilities will be related to the learning sets which must be acquired, in the sense that they will have been identified as the "bottom row" of the learning set hierarchy arrived at by means of the analysis just described; others will be not so related to the hierarchy. In order to have a distinctive set of terms, we may call the former "relevant" basic abilities and the latter "irrelevant." In the sense we wish to use the term, relevant means related by theoretical prediction.

In summary then, the suggestion derived from this theory is that differences in rate of completion of a learning program are primarily dependent upon the number and kind of learning sets (i.e., the "knowledge") the learner brings to the situation, secondarily upon his standing in respect to certain relevant basic abilities, and not in any direct sense upon a general "learning rate" ability.

Statement of the Problem

As is evident from our previous statements, we recognize the possibilities of individual differences at the beginning of a learning program to be: (*a*) differences in knowledge, i.e., the number and pattern of learning sets, both relevant and irrelevant to the final performance; (*b*) differences in amounts of basic abilities, relevant and irrelevant; and (*c*) differences in a general learning ability, general intelligence. Looked at as a whole, the problem may be stated in the following way: what are the causes of individual differences in performance on a learning program, and what relative weighting of causal effect can be assigned to each of them? The implication of the theory previously stated is that a substantial proportion of the variance in learning program performance is attributable to the attainment or nonattainment of learning sets relevant to the final task which the program is designed to teach.

In more specific terms, the theory together with assumptions derived from other psychological findings, would predict the following things about individual differences in programed learning (or, more generally, in productive learning):

1. Individual differences in those beginning a learning program may be independently measured as differences in (*a*) general intelligence, (*b*) relevant basic abilities, (*c*) number and pattern of relevant learning sets. The word "relevant" means "rationally derived as related" in accordance with the analysis procedure previously outlined.

2. An ideally effective learning program has the effect of reducing the variance attributable to 1*c* to zero, since in such a program all learning sets are attained by everyone. To the extent that a learning program is ineffective, however, an increasing number of individuals will be "eliminated" from attainment of learning sets at progressively higher levels of the hierarchy (and accordingly, from attainment of the final task).

3. Both Factors 1*b* and 1*c* are considered to mediate specific, rather than general, positive transfer to the learning of relevant learning sets in the hierarchy. Positive transfer in this situation may be measured by rate of learning, that is, by the time taken by the learner to attain any or all of the relevant learning sets including the final task. Accordingly, as the learner progresses upwards in the hierarchy, his rate of learning should depend increasingly on the attainment or nonattainment of relevant learning sets, and decreasingly on relevant abilities. Specifically, this means that the rate of learning of learning sets will correlate to a decreasing extent with relevant abilities as

one progresses to higher and higher learning sets. Since the same progression partakes increasingly of new learning (as opposed to recall of previously acquired learning sets), there should also be *increasing* correlation of rate of learning with attainment of immediately subordinate learning sets.

4. The effects of Factor 1*a*, general intelligence, should be apparent in moderately low correlations of learning rate with measures of this factor. However, the amount of "general" transfer will be expected to *remain constant* as the learner progresses upwards in the hierarchy, as exhibited by a constant size of correlation coefficient with learning rate at all levels of the hierarchy. The same pattern of correlation (i.e., a constant one) would be expected between learning rates and irrelevant basic abilities, presumably because of the extent to which these too sample general intelligence and mediate general transfer.

5. The relations of basic and general abilities to *achievement* of learning sets in the hierarchy, as opposed to learning rate, appear somewhat more difficult to predict. By achievement here is meant being successful or unsuccessful on the task which represents a learning set, more or less independently of time (within reasonable limits). Primarily because of the reasoning outlined in Paragraph 2, more and more individuals would be expected to "drop out" at higher levels of the hierarchy, to the extent that the learning program is ineffective. As this happens, the "selected" group will presumably contain increasing numbers of individuals who score high on relevant basic abilities (and perhaps to a lesser extent, high on general intelligence). Consequently, correlations of relevant basic abilities with achievement of progressively higher learning sets should exhibit an increasing pattern, in a moderately ineffective learning program. In a sense, then, the rate and amount of such increasing correlation may be looked upon as an inverse measure of the effectiveness of a learning program.

It was our intention to test as many as possible of these predictions from the theory of learning set hierarchies. To do this, we first analyzed a final task represented by an existing learning program in solving linear algebraic equations, to define a hierarchy of learning sets. Before administering the learning program to a group of seventh grade children, we obtained measures of those basic abilities which were revealed by this analysis to be relevant, as well as of two which were irrelevant. During the administration of the program, records were obtained which provided measures of rate of learning of each learning set in the hierarchy. Following this, measures were made of achievement of each learning set, performance of the final task of equation solving, and transfer indicated by performance in solving equations of unfamiliar form and content. Altogether, these measures enabled us to obtain evidence on transfer of training within the determined hierarchy of learning sets, and on the relation of both basic abilities and learning sets (knowledge) to rate of learning, achievement, and transfer within the area defined by the learning program.

Method

Analysis of the Task

The final task for which learning was intended was solving simple linear algebraic equations, either for numerical values of a stated variable, or for expressions of a stated variable in terms of other variables. Examples of these tasks may be found in Appendix B.

Analysis of this task was begun by formulating an answer to the question: "What would an individual have to know how to do in order to achieve successful performance of this class of task, assuming he were given only instructions?" The phrase regarding instructions needs some explanation (Gagné, in press), which may be restated here briefly. The individual must be: told the form of the answer (in this case, whether numerical or symbolic); informed of any definitions of stimuli required; and provided with guidance suggesting the application of previously acquired learning sets to a new task.

Assuming this kind of instruction, the answer to the question "What would the individual have to know?" turned out to be three distinct learning sets (tasks). The first was "simplifying an equation by adding and subtracting *terms*"; the second was "simplifying an equation by multiplying, dividing, adding, or subtracting arithmetic numbers"; and the third was "simplifying an equation by multiplying and dividing by *terms*." The concrete meanings of these descriptions may be grasped by examining the tasks used to measure them, listed as I1, I2, and I3 in Appendix A. What they represent are three types of problem situations which the solver of equations meets in the course of arriving at a solution to equations of the total class. (Of course, there are instances in which the simplifications described would actually lead to a solution, but these are too simple by themselves to represent the total class of linear equation problems.) Each of these tasks is distinct: that is, it is possible to know how to do any one without knowing how to do the others. Each, as will become apparent, is based upon different patterns of subordinate knowledge.

This analysis was then repeated on each of the three learning sets defined as subordinate to the task (Level I), until the entire hierarchy was defined as shown in Figure 1. In this figure, each block contains a description of a learning set. The large dots under each block indicate which of the lines leading from subordinate learning sets is "tied into" the learning set defined in that block. Sometimes, a subordinate set several levels down the hierarchy is related to a learning set in this manner; in such instances, the line depicting the relationship may pass underneath an intervening block, and the absence of a dot indicates no "tie-in" with this intermediate learning set. The learning sets are arranged into levels indicated by roman numerals, primarily for the purpose of denotation. No particular significance is attached to these "levels," except that the learning sets on any given level appear to be approximately equivalent in complexity. In referring to a particular learning set, one may use the roman numeral to designate level and arabic numerals to denote its horizontal position beginning with 1 at the left. Thus, the three learning sets previously described are I1, I2, and I3.

In carrying out the analysis, we were of course guided by the *outline* of the learning program which was to be used in the study. This does not mean, however, that we derived the learning sets in the hierarchy directly from the program—quite the contrary was the case, and we shall point out later that our method of analysis revealed two learning sets which were inadequately represented in the program. What it does mean is that in doing the analysis we accepted in general the approach to solution of simple equations which had been designed into the program. This particular approach is by no means the only one. In other words, there are perhaps several *possible* learning set hierarchies which could be worked out to support this final task, and it is quite conceivable that some are "better" than others in the sense of being more efficient or more transferable to later learning. The proponents of instruction in "modern mathematics" would almost surely not favor this particular one.

As the theory foresees, the successive application of the analysis procedure defines learning sets which are *increasingly simple* and *increasingly general* in the sense that they are potentially supportive of greater numbers of superordinate learning sets. A learning set like IIIA2 (simplifying fractional expressions), for example, must support quite a large number of different tasks in the field of arithmetic. The learning sets at Level IVA are seen to be almost as simple as can be defined, for a human being. The concrete form of each learning set may be understood by referring to Appendix A.

Somewhat parenthetically, it needs to be stated that the descriptions of learning sets employed in Figure 1 employ somewhat old-fashioned language, judged against the standards of modern algebra texts. The language of the learning program was similarly old-fashioned. The validity of the procedure, however, should not be questioned on this account. When the learning sets are understood as tasks, which can be done by examining the problems given in Appendix A,

the ambiguities of language are largely removed, and some intuitive grasp of the relatedness of the various learning sets in the hierarchy may be gained. It may also be emphasized that our purpose in this study was to test a method and some theoretical deductions which presumably are applicable to *any* task and *any* learning program. We

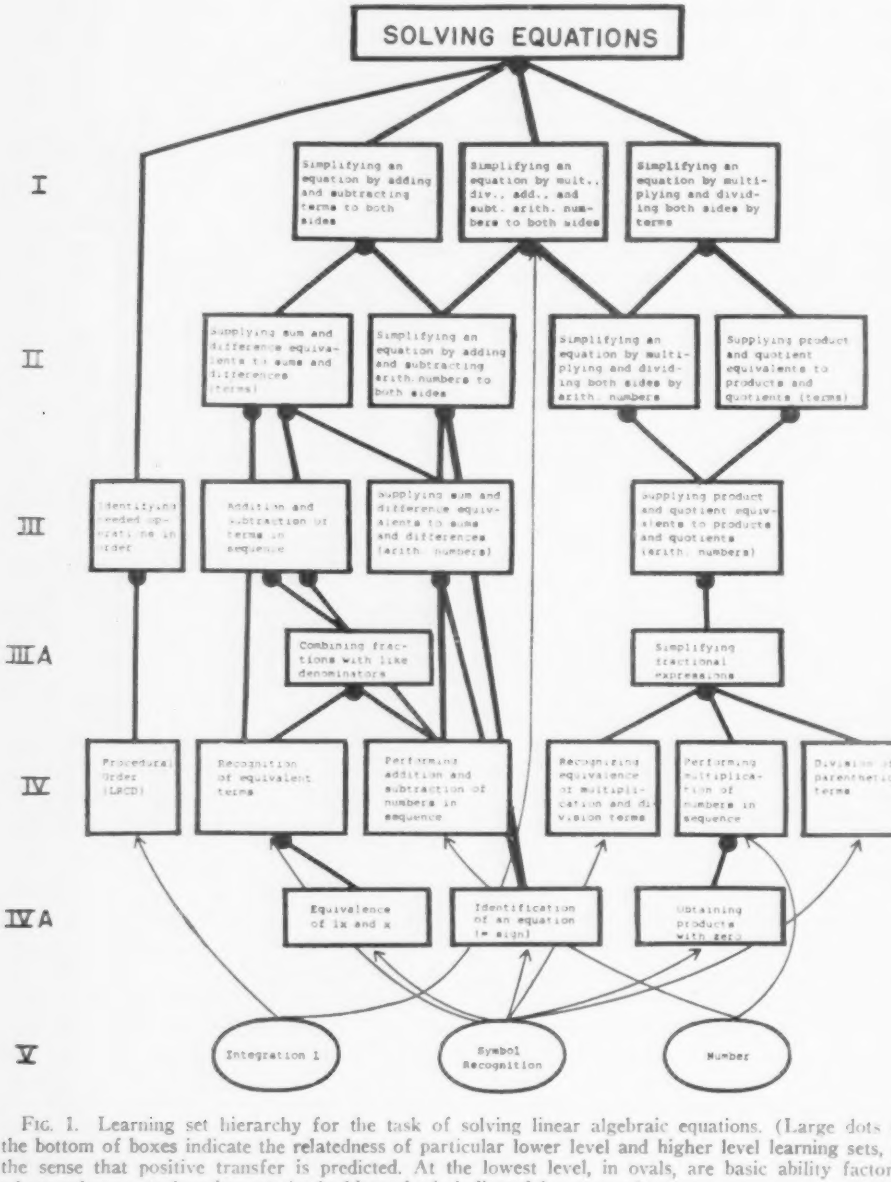


FIG. 1. Learning set hierarchy for the task of solving linear algebraic equations. (Large dots at the bottom of boxes indicate the relatedness of particular lower level and higher level learning sets, in the sense that positive transfer is predicted. At the lowest level, in ovals, are basic ability factors, whose relevance to learning sets in the hierarchy is indicated by arrows.)

were not directly interested in the effectiveness of the particular program we employed, nor in the modernity of its content.

Level 5 of the hierarchy defines tasks which we hypothesize to be equivalent to those occurring in "factor reference" tests (French, 1954). Number is the name we use for tasks requiring the arithmetic operations of addition, subtraction, multiplication, and division of one- and two-place numbers. Symbol Recognition is a variety of Associative Memory usually measured by such a test as Picture-Number (French, 1954, p. 15). Beyond these two familiar factors, we found it necessary to identify another which is not so well known, called Integration I (Guilford & Lacey, 1947; Lucas & French, 1953). In its simplest form, it appears to be the capability of "holding in mind" several operations in sequence, and is often measured by tests of following directions. It should be emphasized that these capabilities (the learning sets at Level V) were derived by the same analysis procedure as were others in the hierarchy. We did not *set out* to find tasks which resemble those used in measuring basic factors. Instead they represent the simplest kinds of things an individual must "know how to do" in order to progress up through the hierarchy. Having identified them, however, it seemed important to gather evidence which would relate them as basic abilities to performances of higher level tasks.

Materials

Tests of basic abilities. To obtain measures of basic abilities, we used factor reference tests suggested by French (1954). Number Ability was measured by the two tests Addition, and Subtraction and Multiplication. Symbol Recognition ability (Associative Memory) was measured by means of the test Picture-Number. Integration was measured by a test called Following Directions.[3] In addition, to measure irrelevant abilities, we used two which would provide a rather severe test of our hypothesis, Verbal Knowledge (measured with a test called Vocabulary, V-1) and Speed of Symbol Discrimination (Letter A).[4]

Learning program. The learning program designed to teach the solving of linear equations was originally composed of 247 frames. These were printed on 4″× 6″ cards, and were divided into eight convenient and roughly equal sequences, which were assembled into booklets with plastic hinges at the top. The new frames in each of the Booklets 2 through 8 were preceded by several review frames (5 to 17 in number) identical to certain frames in the previous booklet. Thus each booklet contained from 36 to 45 frames. The eight booklets were designed to be used in eight class periods over eight successive school days. The answer to each item on the front of the card was printed on the back. Mimeographed answer sheets, with blanks numbered to correspond to frame numbers, were used for the recording of answers by the students.

Performance measures. Designed to be administered after the completion of the learning program were three performance measures. The first was a *performance test* of equation solving, containing 10 simple linear equations of the sort encountered in the learning program (see Appendix B). This test had a time limit of 25 minutes. Scoring allowed for partial correct completion of problems, on a scale of 0-4 for each, making the maximum possible score 40. The second measure was a *transfer test* (Appendix B), containing 10 additional linear equations having somewhat unfamiliar forms and unfamiliar symbols. This was a 20-minute test, and was scored in a manner similar to that used with the performance test. The third test was one designed to measure achievement on each of the 22 learning sets identified by the analysis previously described. As administered, a single item was used to measure each learning set, and a total class period of 50 minutes was allowed for completion. These items are given in Appendix A.

Subjects

The subjects were members of four different mathematics classes, two in the seventh grade of Kensington Junior High School, and two in the seventh grade of Montgomery Hills Junior High School, both located in Montgomery County, Maryland. The classes contained students of intermediate abilities, heterogeneously grouped. The data of students who missed one or more of the sessions involved in the study were eliminated from consideration in the results. Of the 144 students in the four classes, 26 missed at least one of the sessions, leaving an N of 118.

Procedure

The administration of the tests and learning program was carried out by the experimenter in each classroom. The classroom teacher remained in the room, for the most part, during these

[3] Supplied by John W. French, Educational Testing Service, and used with permission.

[4] Used with the permission of the copyright owner, Thelma G. Thurstone.

sessions. The teacher made assignments of work for students to undertake, at their desks, once they had completed each booklet of the learning program. These arrangements were made before each session began. The assignments were composed of unrelated materials.

During the first class session for each group, the tests of basic abilities were administered, following the directions given by French (1954). The learning program was then administered, using the eight booklets, on eight successive class days. Three school days intervened between the administration of Booklets 6 and 7, owing to the interruption of school attendance by a severe snowstorm. On the day following completion of the learning program, the performance test was administered in 25 minutes, followed by the transfer test for 20 minutes. On the following day, the test of learning sets was given and the total period of 50 minutes allowed for its completion. Actually, there were 44 items on this test, 2 for each learning set, but since it was determined that these could not be completed, the students were instructed to do each odd-numbered item before going on to even-numbered ones, and the former scores were used in obtaining the measure of learning set achievement.

In administering the learning program, the experimenter first read through the instructions which appeared on the first two cards of Booklet 1. These told the students how to use the booklets and how to record their answers. Students were instructed to write an answer to the question asked in each frame, and then to flip the card over and compare their answer with the correct answer printed on the back of the card. If their answer was right, they were to proceed; if wrong, they were to draw a line through it, turn the card back again until they could see how the right answer was obtained, and then record the correct answer and proceed.

At the beginning of each learning session on successive days, the experimenter repeated the main points of these instructions. The students were especially cautioned to record each answer before flipping over the card to check the answer. (This behavior was monitored throughout by the experimenter.) They were asked to be certain that they knew the material on each card before proceeding with the next one.

During the administration of each booklet of the program, students were instructed to draw a line at the edge of their answer sheets under each question they had just completed, each time the experimenter called "Mark!" This signal was given every 3 minutes, although the students were not told the amount of this interval. These records were used to compute learning rate of each section of the program. A score of learning rate was later obtained for each learning set by adding the times taken to complete the frames devoted to that learning set, interpolating where necessary. Since the program devoted only one frame to each of the Learning Sets III1 and II4, no learning rate score was obtained for these.

RESULTS

The results of the study may be described in accordance with the following organization. First, we shall be interested in reporting the several measures of learning and performance that were obtained, which provide a general picture of the outcomes of administration of the learning program. Second, a matter of major interest is the evidence of transfer among learning sets in the hierarchy shown in Figure 1. Do results indicate that these learning sets mediate positive transfer to higher level learning sets, as proposed by the theory? Third, what correlations were found among relevant and irrelevant basic abilities and total measures of performance during and after the completion of the learning program? And finally, there is particular interest in seeing whether patterns of increasing and decreasing correlations are obtained, between abilities on the one hand and rates of attaining individual learning sets on the other.

Measures of Learning and Performance

The several measures of learning and performance applicable to the entire program are shown as means with associated standard deviations in Table 1.

TABLE 1

MEANS AND STANDARD DEVIATIONS OF SEVERAL MEASURES OF LEARNING AND PERFORMANCE APPLICABLE TO THE ENTIRE LEARNING PROGRAM

($N = 118$)

Measure	*M*	*SD*
Performance on equation solving test (10 items, 40 points)	8.4	4.6
Transfer performance in solving unfamiliar equations (10 items, 40 points)	6.1	3.7
Total number of learning sets achieved (22 items)	12.4	3.4
Time to complete program (learning rate) in minutes	221.4	22.4

It may be seen that in terms of performance in equation solving, the learning program was not very successful. This is also indicated by the scores on the transfer test containing 10 equations of somewhat unfamiliar form and symbolic content, scored with a maximum of 40 points. The mean number of learning sets achieved, as measured by a test administered following the program, was 12.4, another indication of the moderately low effectiveness of the program.

Measures of Transfer among Learning Sets

It will be recalled that the theory predicts high positive transfer from a recalled learning set (or sets) at a given level and attainment of the adjacent higher relevant learning set(s). This prediction can be tested by noting the pattern of pass and fail which obtains between the lower and higher adjacent sets throughout the hierarchy. It may be noted that where two or more subordinate learning sets are involved (for example, as III1 and III2 are related to II1, Figure 1), the prediction is that *all* lower relevant sets must be passed, in order for the higher level set to be attained.

The four possible empirical relationships for passing and failing relevant higher-lower learning set combinations are as follows, together with the theoretical significance of each:

1. Higher+, Lower+: This relation indicates the occurrence of positive transfer from lower learning set(s) to adjacent higher learning set(s), and is in accord with the theory.

2. Higher−, Lower−: If any relevant adjacent lower level set has been failed, positive transfer to the higher level learning set is unlikely; this outcome is also in accord with the theory.

3. Higher+, Lower−: As indicated in 2, this outcome is directly opposed to the theoretical prediction.

4. Higher−, Lower+: When this outcome takes place, it is not in opposition to the theory. A higher level set may be failed, even though all relevant lower level sets are passed. A number of reasons make this occurrence possible, most of them associated with the effectiveness of the learning program.

The patterns of pass-fail relationships indicated by data obtained on the test of learning set achievement administered following the learning program are given in Table 2.

Each of the transfer relationships which can be measured between learning sets in the hierarchy is listed in the first column of this table. As would be expected with a program that is not perfectly effective, the number of ++ relationships between higher and lower learning set combinations goes steadily down as the learning program carries the learner upward in the hierarchy, whereas the number of −− relationships increases. Instances of higher +, lower − are cases of positive transfer without the achievement of lower level learning sets, and are therefore contrary to theoretical prediction. It is noteworthy that such instances are very low in frequency. The proportion of pass-fail patterns which support the theory, obtained by dividing the total testable instances (++, −−, +−) into the number of instances consistent with the hypothesis of positive transfer (++, −−) is shown in the final column. Some of these proportions are 1.00, and none is lower than .91.

The theoretical prediction is for the values in the final column of the table to be 1.00. Obviously, they are very nearly that, and far above the purely chance values of these patterns, which would range between .25 and .50. One reason, irrelevant to theory, for their departure from the 1.00 value is of course unreliability of the measurements from which they are derived. It will be recalled that the measures employed are single item pass-fail scores. The use of two or more items to assess learning set attainment would have made possible an estimate of the reliability of these measures, but unfortunately these were not used in the present study. In future studies, provision should surely be made for reliability measures. At any rate, on the basis of present evidence the prediction of positive transfer

TABLE 2

PASS-FAIL PATTERNS OF ACHIEVEMENT BETWEEN ADJACENT LOWER AND HIGHER LEVEL RELEVANT LEARNING SETS, AND THE PROPORTION OF INSTANCES OF POSITIVE TRANSFER INDICATED ($N = 118$)

Transfer to learning set	Frequency of pass-fail Pattern—Higher, Lower				Total testable frequency (1) + (2) + (3)	Proportion positive transfer
	++ (1)	−− (2)	+− (3)	−+ (4)		
IV2 from IVA1	110	0	0	8	110	1.00
IV5 from IVA3	113	0	0	5	113	1.00
IIIA1 from IV2, IV3	85	0	7	26	92	.92
IIIA2 from IV4, IV5, IV6	94	5	10	9	109	.91
III1 from IV1	45	9	1	63	55	.98
III2 from IV3, IIIA1	68	30	6	14	104	.94
III3 from IVA2, IV3	75	25	7	11	107	.93
III4 from IIIA2	62	40	4	12	106	.96
II1 from IV2, III2, III3	34	70	3	11	110	.95
II2 from IVA2, III3	41	60	2	15	103	.98
II3 from III4	37	72	3	6	112	.97
II4 from III4	9	85	0	24	94	1.00
I1 from II1, II2	25	78	2	13	105	.98
I2 from II2, II3	28	80	3	7	111	.97
I3 from II3, II4	6	104	0	8	110	1.00

mediated by learning sets appears to be amply supported.

The values for the −+ pass-fail pattern deserve some comment. As has been said, these are not critical evidence for or against the theory. What they do indicate, presumably, is relative weakness in portions of the learning program. For example, 63 of the 118 learners failed to progress from Learning Set IV1 to Learning Set III1, after having *achieved* the former adequately. Examination of the program shows that this particular set (III1, identifying needed operations in order) was covered by only a single frame. A similar finding holds for Learning Set II4, which has 24 failures after success at a lower level. Learning Set IIIA1, with 26 failures following lower level successes, is represented by only two frames. The occurrence of these instances suggests strongly that the values in the −+ column of Table 2 indicate the points at which greater or lesser effectiveness was attained within the total learning program. High values may be interpreted as revealing the points at which learning was relatively ineffective, low values the points at which learning was effective.

The measurement of learning sets makes it possible to assess in a fairly exact fashion what the *individual* has learned or has not learned from a learning program. Examples of the patterns of learning set attainment in a low achiever and a high achiever are shown in Figure 2.

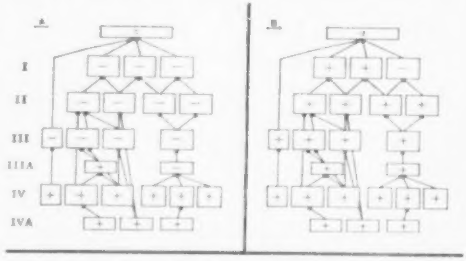


FIG. 2. Examples of patterns of attainment of learning sets in the hierarchy for two individuals, a low and a high achiever. (Scores on the final performance test of equation solving are indicated for each individual in the box denoting the final task.)

Basic Abilities and Measures of Performance

Product-moment correlations between the basic ability measures and four total performance measures are given in Table 3.

On the whole, these coefficients yield an expected pattern. Particularly notable is the fact that correlations between each of the basic variables Number, Symbol Recognition, and Integration (measured by tests numbered 1, 2, and 3) and each of the four performance measures (6, 7, 8, and 9) are moderately high. It will be recalled that these three abilities are considered to be relevant. In contrast, correlations of the irrelevant ability measured by Vocabulary with these performance variables are fairly low, despite the fact that this test is presumed to be highly related to general intelligence. It will also be noted that Letter A exhibits higher correlations with performance measures than would be expected of an irrelevant ability. Although correlations with this test are all slightly lower than those for the relevant abilities, nevertheless these results are not entirely in accord with our predictions. Of course, it is possible that speed of Symbol Discrimination is, after all, a relevant basic ability; but our analysis did not predict it to be so.

Correlations among the performance measures themselves are fairly high. In general, individuals who did well on the final test of performance were those who mastered the greatest number of learning sets, and took the shortest time to complete the program. Achievement on the final test is also highly related (.84) to achievement on the transfer test; it appears likely that these tests are measuring the same capability.

Relations between Abilities and Learning Sets

Point-biserial coefficients of correlation were obtained between each basic ability measure (relevant and irrelevant) and pass-fail achievement of each learning set as measured following learning. Product-moment coefficients of correlation were obtained between each basic ability measure and rate of learning (time to complete the required frames, with sign reversed) for each learning set in the hierarchy.[5] For both sets of data, interest centered in the pattern of increasing or decreasing correlation which might be revealed as one considers the learning sets from the bottom of the hierarchy upwards. It will be recalled that the theory predicts: decreasing correlation of relevant abilities with rates of attaining learning sets

[5] Complete data on intercorrelations of these variables are given in Appendix C.

TABLE 3

PRODUCT-MOMENT COEFFICIENTS OF CORRELATION AMONG TESTS OF BASIC ABILITIES ADMINISTERED BEFORE LEARNING AND FOUR MEASURES OF PERFORMANCE

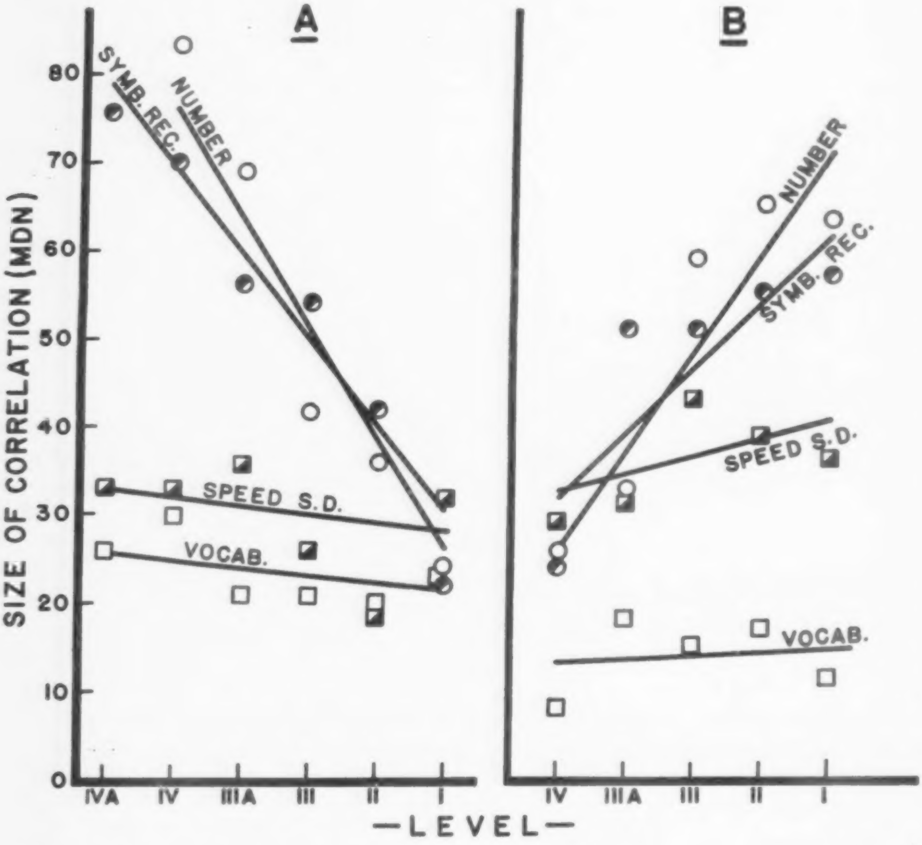
($N = 118$)

Measure	1	2	3	4	5	6	7	8	9
1. Addition, Subtraction-Multiplication	91[a]	41	33	04	38	68	66	58	55
2. Picture-Number		77[b]	39	14	34	62	58	56	54
3. Following Directions			—[c]	12	34	58	54	53	52
4. Vocabulary V-1				94[b]	05	22	14	12	18
5. Letter A					81[b]	51	45	46	50
6. Performance test						—	84	82	78
7. Transfer test							—	78	75
8. Number of learning sets achieved								88[b]	82
9. Time to complete program (sign reversed)									84[b]

Note.—Reliabilities shown in diagonal.
[a] Correlation between Addition and Subtraction-Multiplication.
[b] Odd-even split-half correlation.
[c] Unavailable.

as one progresses upwards; increasing correlation of relevant abilities with achievement of learning sets, proceeding in the same direction; and moderately low and constant correlations of irrelevant abilities (and general intelligence) with both learning rate and achievement.

Examination of Figure 1 will reveal how the relevant learning sets were selected for this treatment of the data. For Symbol Recognition ability, the relevant learning sets begin with IVA1, 2, and 3; at Level IV, however, only Learning Sets IV2, IV4, IV5, and IV6 are relevant (IV1 and IV3 are connected to Symbol Recognition neither by an arrow nor by an intermediate learning set at Level IVA). At higher levels, all the learning sets are relevant to Symbol Recognition except for III1, since they are connected to this ability via intermediate learning sets. For Number Ability, relevant learning sets begin with IV3 and IV5; again, all



3a. The relationships with learning rate, based upon product-moment correlations.

3b. The relationships with achievement, based on point-biserial coefficients.

FIG. 3. Median values of coefficients of correlation of relevant basic abilities (Number, Symbol Recognition) and irrelevant basic abilities (Speed of Symbol Discrimination, Vocabulary) with measures of learning sets at different levels of the learning set hierarchy. (For irrelevant sets, medians of the highest three coefficients are shown, except for Level IIIA which contains only two. In all cases, straight lines have been fitted to the sets of points by eye.)

others are relevant except for III1. For Integration, the only relevant sets are IV1, III1, and I2, and these data were not plotted because of the sparsity of points which could be determined.

Figure 3 graphs the relationship of the size of the median correlation obtaining between relevant basic abilities and learning sets and between irrelevant basic abilities and learning sets, to the level of the hierarchy in which the sets are located. Figure 3a pertains to learning rate; Figure 3b, to achievement of learning sets. In order to provide an idea of the slope of the relationships, straight lines have been fitted to the points by eye. The median correlation at each level was determined from values of the particular number of relevant learning sets at that level. For the irrelevant sets, the median of the *three highest* correlations was taken (except for Level IIIA, where only two exist), in order to have the number of values on which to base a median comparable to those for relevant sets.

Considering the data shown in Figure 3a, it is apparent that there is quite a marked difference between the changes that occur in the correlations between relevant sets and learning rate, and those between irrelevant sets and learning rate, as one progresses upwards in the hierarchy. Relevant abilities are correlated highly at low levels of the hierarchy, and markedly decrease in correlation at high levels. In contrast, the changes in correlation with irrelevant abilities, while not absent, exhibit an extremely small slope. These results, then, tend to confirm the theoretical prediction that rate of learning depends *decreasingly* upon relevant abilities as learning progresses upwards in the hierarchy. In addition, irrelevant abilities exhibit moderately low and nearly constant values of correlation with the rate of attaining learning sets.

The picture for correlations of abilities with achievement of learning sets (Figure 3b) is nearly the opposite in every respect. Here, there is a pattern of *increasing* correlation between relevant abilities and learning set achievement. The explanation of this trend is not crucially dependent on theory in this case, but is based on the reasoning that increasing numbers of individuals effectively "drop out" as learning proceeds; thus the variance which remains becomes more clearly that of relevant basic abilities. Again, in the case of irrelevant abilities, the change in the size of correlation is not nearly so great. The smallest slope is obtained for Vocabulary, often considered to be a measure of general intelligence, while Speed of Symbol Discrimination attains an intermediate slope. On the whole, the contrast between the behavior of relevant and irrelevant sets is a marked one, nearly as striking as that which obtains for learning rate (Figure 3a), but in the opposite direction.

The correlations obtained between learning rate and Integration were .76 with IV1 and .43 with I2; it will be recalled that the learning rate of III1 could not be measured. This test, too, shows the decreasing pattern found with other relevant abilities. For achievement of learning sets, the values of the correlations were .53, .50, and .56 at Levels IV, III, and I, respectively. These data do now show the kind of increasing pattern which obtains for the other relevant abilities.

Relations among Learning Sets

If the rate of learning for progressively higher learning sets in the hierarchy depends decreasingly upon relevant basic abilities, then, according to the theory, it must come to depend increasingly upon *attained knowledge,* that is, on the successful achievement of relevant subordinate learning sets. Accordingly, the data were next examined for evidence of this latter relationship.

Table 4 contains median point-biserial coefficients of correlation for relationships between achievement of each learning set and rate of learning of *relevant* adjacent higher learning set or sets. These are compared in each case with correlations between achievement of the same learning set and *irrelevant* adjacent higher learning sets.[6] The table indicates the adjacent higher sets in each instance from which the median

[6] The individual correlation coefficients are given in Appendix C.

correlation was derived; the distinction between relevant and irrelevant sets was of course derived from the theoretically predicted connections shown in Figure 1. (This analysis of the data begins at Level IV, since achievement of learning sets at Level IVA was virtually perfect, and thus provided no variance.)

Generally speaking, the correlations of achievement and learning rate for relevant pairings show a moderate rise as one goes upwards in the hierarchy. In addition, each of the correlations for relevant pairs is higher than the corresponding correlation for irrelevant pairs. Tests of significance of these differences (utilizing the median intercorrelations shown in the next-to-last column to obtain a value of t applicable to correlated samples) are shown in the final column of the table. Half of these comparisons attain a satisfactory level of significance, while half do not. At the higher levels of the hierarchy, all the comparisons are significant ones, indicating that learning rate at these levels is dependent upon achievement of adjacent subordinate learning sets. This finding is to be contrasted particularly with the previous one concerning the relations between basic abilities and learning rates; it will be recalled that for these variables, relationships pertaining to relevant ones become indistinguishable from irrelevant ones at higher levels of the hierarchy. It does appear, therefore, that rate of attainment of learning sets comes to depend increasingly upon *knowledge* achievement (as indicated by successful achievement of adjacent subordinate learning sets), and decreasingly upon amounts of basic abilities.

Discussion and Implications

The results have indicated, first of all, that the acquisition of individual capability in solving linear equations, established by a learning program, may be conceived as a matter of attaining a hierarchy of learning sets which may collectively be called knowledge. A final task of this sort may be analyzed to reveal a supporting hierarchy of learning sets by asking the question: "What would an individual have to know how to do

TABLE 4

Median Coefficients of Correlation (r_{pbi}) of Relevant and Irrelevant Pairings Comprising Learning Set Achievement vs. Learning Rate of Adjacent Higher Learning Set(s), with Significance of Differences between These Coefficients

($N = 118$)

Achievement on learning set: (1)	Relevant pairing: Learning rate of: (2)	Mdn. r_{pbi}	Irrelevant pairing: Learning rate of: (3)	Mdn. r_{pbi}	Mdn. r_{pbi} (2) vs. (3)	p of difference (r_{12}-r_{13})
IV2	III1; IIIA1	43	II2, 3; IIIA2	25	30	>.05<.10
IV3	III2, 3; IIIA1	44	III4	19	25	<.05
IV4	IIIA2	34	IIIA1	18	26	>.10
IV5	IIIA2	33	IIIA1	19	26	>.10
IV6	IIIA2	35	IIIA1	21	26	>.10
IIIA1	III2	32	III3, 4	21	29	>.10
IIIA2	III4	35	III2, 3	22	28	>.10
III2	II1	38	II2, 3	22	40	>.05<.10
III3	II1, 2	42	II3	18	29	<.02
III4	II3	41	II1, 2	19	29	<.05
II1	I1	55	I2, 3	30	37	<.01
II2	I1, 2	54	I3	25	36	<.01
II3	I2, 3	49	I1	24	37	<.01
II4	I3	57	I1, 2	30	36	<.01

in order to perform this task, after being given only instructions?" The answer to the question defines immediately subordinate learning sets, and the question may then be applied to these tasks in turn to define the next subordinate level of learning sets. By theory, each learning set is conceived to function by mediating positive transfer to a higher level task or tasks.

In this study, such an analysis defined 22 subordinate learning sets arranged in hierarchical fashion. When the question described previously was applied to those sets occurring at the next to the lowest level, the answer defined the additional tasks of Number, Symbol Recognition, and Integration, which appeared to be the same as those used in certain factor-reference tests.

Relationships among the variables measured in the study lead to the conclusions that: (*a*) a high incidence of positive transfer (nearly 100% of the instances tested) obtains from success on relevant subordinate learning sets to attainment of a superordinate learning set; (*b*) a decreasing pattern of correlations can be shown between relevant basic ability factors and rate of learning for learning sets as one progresses upwards in the hierarchy; (*c*) in contrast, correlations of rate of learning with irrelevant ability factors show only a slightly decreasing trend; (*d*) learning rate of the learning sets in the hierarchy depends increasingly on acquired knowledge (in the form of subordinate learning sets) and decreasingly upon basic ability factors as one proceeds from the bottom to the top of the hierarchy.

Implications for Ability Measurement

These findings may have important implications for the measurement of abilities involved in any human task. For one thing, they emphasize the importance of measures of rate of learning as criteria against which the predictive efficiency of an aptitude test may be assessed. There appears to be a fairly clear rationale for relationships between an initial ability score and rate of learning of a relevant task, if one assumes that the ability represents a level of capability which mediates a greater or lesser amount of positive transfer. Basic abilities, or factors, may accordingly be conceived as the *most simple* and *most general* learning sets, which support a great variety of more complex activities. Such support manifests itself as greater or lesser facilitation of the rate of learning or related tasks.

The suggestion is, therefore, that a true integration of measures of productive learning and aptitude measures can be effected by identifying and correlating *rate of learning of learning sets* in a hierarchy with basic ability factors that are defined by means of the same analysis. Such a procedure would make possible the *testing of hypotheses* in aptitude studies, in much the same fashion as has been done in the present study. It would also reveal a rational basis for the statement that human tasks "depend upon" or "are related to" aptitude factors. And particularly, such studies should make possible a suitable determination of the relative contribution of knowledge variables, as compared with basic ability variables, in the achievement of performance on a final task (representing a class of tasks).

The correlation of basic ability factors with achievement, our results suggest, is really a more complex matter for which the rationale is less clear. Specifically, the achievement of any given task (which may be any learning set in a hierarchy) depends not only upon the amount of basic ability, but also upon the amount and kind of specifically transferable knowledge that has been acquired. As learning proceeds to higher level learning sets, relevant abilities will correlate to an increasing extent with achievement *only to the extent that the learning program is ineffective* in mediating transfer from one level of the learning set hierarchy to the next. A "power" test of achievement, containing items of increasing complexity, may be conceived as a deliberately inefficient learning program, whose correlation with basic ability factors depends upon the fact that increasing numbers of people "drop out" as they attempt more and more items of the test. This is, of course a perfectly good rationale for such a test. But it does fail to distinguish between the contributions made to performance by knowl-

edge factors as opposed to ability factors. For precise prediction of performance on an achievement test, one must measure the recallability of subordinate learning sets, rather than search for additional basic abilities.

Learning Programing

As has been pointed out in a previous article (Gagné, in press) the theory of learning set hierarchy has a number of implications for the programing of productive learning. Chief among these is the idea of designing the frames of a program in such a way that they: constitute an ordered sequence logically related to the hierarchy of learning sets for the desired final task, provide for recallability of subordinate learning sets, and furnish the guidance to thinking which will enable the learner to integrate subordinate learning sets in the performance of new tasks. The learning program used in the present study was not constructed according to these principles. Nevertheless, the evidence has some things to say about them.

Although the program was not deliberately designed to establish the learning sets identified by theoretical analysis, the evidence indicates that this is what it did do, to the extent that it was effective. Furthermore, there is good evidence that when the particular learning sets required for new learning were present in the individual, high positive transfer resulted; when they were absent, very low transfer took place (cf. Table 1). It is also significant to note that the analysis, carried out according to theory, was able to identify three learning sets which were inadequately emphasized within the program (Learning Sets IIIA1, III1, and II4, Figure 1) in the sense that only one or two frames was devoted to each. The evidence suggests that transfer was particularly low at just these points in the program. It seems reasonable to conclude that the analysis of a final task into subordinate learning sets can be an important, if not essential, first step in the development of an effective learning program.

The use of learning set analysis to identify the required sequence of a learning program, and later to measure its effectiveness, can provide a solution to a measurement difficulty encountered by methods in common use at present. On logical grounds, internal criteria of program effectiveness such as time to complete frames or number of errors per frame, cannot be considered convincing evidences of how well a program is accomplishing what it is designed to do. Measures of final performance or of transfer are beyond doubt best suited for this purpose. However, the latter measures are not particularly *diagnostic* of the relative effectiveness of portions of the learning program. The measurement of learning sets has the advantage of accomplishing both these purposes: it can diagnose weak points, and it can measure the achievement of the learner up to any point at which he "loses understanding," or effectively "drops out." The evidence of this study shows that the measurement of learning sets can provide the experimenter with relatively precise information about the progress of the individual throughout the course of learning.

Summary

An analysis was made of the class of tasks "solving linear algebraic equations," following the outline of a learning program designed to establish proficiency in these tasks, to identify a hierarchy of learning sets which support the attainment of such proficiency. This analysis was based upon a theory to the effect that attainment of any given learning set is dependent on recallability of certain subordinate learning sets, instructions defining the stimuli and goal of the new task, and integration by the learner of subordinate learning sets into the solution of the new task. Subordinate learning sets are conceived as having the function of mediating positive transfer to higher level learning sets throughout the hierarchy, and ultimately to the final task. Subordinate learning sets for a given class of tasks may be defined as the answer to the question: "What would the individual have to be able to know how to do, in order to be able to perform this (new) task, being

given only instructions?" Beginning with the final task, the question is applied successively to each learning set so defined, and thus identifies a progression of learning sets which grow increasingly simple and increasingly general.

Besides defining a hierarchy of 22 learning sets, three additional ones, represented by very simple tasks, were derived from this analysis at the lowest level of the hierarchy. These appeared to be identical to three tasks occurring in ability tests which have been identified as relatively "stable" in factorial studies, the so-called "factor-reference" tests. Specifically, these tasks appeared to be those involved in tests measuring Number Ability, Symbol Recognition (a particular form of Associative Memory), and Integration I (an ability apparently involving keeping several procedures in mind at once, as in "following directions"). The occurrence of these abilities as an end result of a theoretical analysis of the knowledge composition of a learning program naturally raised the interesting question as to how they function in support of the learning of the final performance, as well as of the intervening learning sets in the hierarchy.

According to theory, basic abilities which are relevant to learning sets in the hierarchy should mediate positive transfer to them, and this in turn should be measurable as an increased rate of learning. In progressing upwards in the learning set hierarchy, correlations of these abilities with rate of attainment of relevant learning sets should *decrease*, since such relations come to depend increasingly upon transfer from immediately subordinate learning sets (i.e., upon specific knowledge). General intelligence, while it may be expected to correlate to a moderate degree with rate of learning throughout the hierarchy, should show *no change* with position of learning sets at various levels of the hierarchy, since it is conceived to mediate general, rather than specific, transfer. The same is true for basic abilities which are irrelevant to the learning sets to be learned, in the sense that they have not been connected to them by analysis. As for correlations of basic abilities with achievement (pass-fail) of relevant learning sets, the prediction would be that these would exhibit an *increasing* pattern as one progresses upwards in the hierarchy. The reasoning is that, to the extent that the learning program is ineffective in reducing all individual differences in achievement, increasing numbers of individuals "fail to grasp" the program as it proceeds; consequently, the relationship comes to depend increasingly on a differentiation of high from low ability. Thus increasing correlations of achievement with relevant basic abilities provide an inverse indication of the effectiveness of the learning program. If the program were perfectly effective, differences in achievement would of course not be measurable.

To test these predictions, a study was conducted by administering the learning program on equation solving to a group of 118 seventh graders in four different school classes. The program was divided into eight booklets, and administered during eight successive class days in the school room. Preceding this administration, factor-reference tests were given to obtain measures of basic abilities, both relevant (Number Ability, Symbol Recognition, Integration I), and irrelevant (Speed of Symbol Discrimination, Vocabulary). During the learning program administration, students were required to mark the margins of their answer sheets at 3-minute intervals, to provide the basis for a measure of rate of learning. Following completion of the program, in another class period, two 10-item tests were administered to measure performance in equation solving, and transfer to the solving of equations having somewhat unfamiliar symbols and form. Finally, a test was given to measure achievement of the 22 learning sets in the hierarchy.

Predictions were confirmed in the following respects:

1. Correlations of theoretically relevant basic abilities were higher than those of irrelevant basic abilities with measures of final performance, with transfer of training scores, with number of learning sets achieved, and with rate of learning of the total program.

2. Instances of positive transfer to each learning set from subordinate relevant learn-

ing sets were found to occur throughout the hierarchy with proportions ranging from .91 to 1.00.

3. Correlations of relevant basic abilities with *rates of attainment* of learning sets at progressively higher levels of the hierarchy showed a steeply progressive decrease. In contrast, the pattern of correlations of irrelevant basic abilities with the rates of attainment of comparable learning sets remained nearly constant.

4. Correlations of relevant basic abilities with *achievement* of learning sets at progressively higher levels showed an increasing pattern, whereas the comparable pattern with irrelevant abilities exhibited at the most a very gentle increase. Correlations of learning set achievement with Integration I (defined as a relevant basic ability) did not reveal an increasing pattern, however.

5. Correlations of rate of attainment of learning sets with achievement of relevant subordinate learning sets were found to be systematically higher than with achievement of irrelevant sets, and significantly so in the upper levels of the hierarchy. This finding confirms the learning set analysis, as well as the idea that rate of learning of learning sets at progressively higher levels of the hierarchy comes to depend increasingly upon specific transfer from subordinate learning sets.

REFERENCES

French, J. W. *Manual for kit of selected tests for reference aptitude and achievement factors.* Princeton, N. J.: Educational Testing Service, 1954.

Gagné, R. M. The acquisition of knowledge. *Psychol. Rev.*, in press.

Guilford, J. P., & Lacey, J. I. (Eds.) Printed classification tests. *AAF Aviat. Psychol. Program res. Rep.* 1947, No. 5.

Harlow, H. F. The formation of learning sets. *Psychol. Rev.*, 1949, **56**, 51-65.

Lucas, C. M., & French, J. W. *A factorial study of experimental tests of integration, judgment, and planning.* (ETS Res. Bull. 53-16) Princeton, N.J.: Educational Testing Service, 1953.

Lumsdaine, A. A., & Glaser, R. (Eds.) *Teaching machines and programmed learning.* Washington, D.C.: National Education Association, 1960.

Skinner, B. F. Teaching machines. *Science,* 1958, **128**, 969-977.

Woodrow, H. The ability to learn. *Psychol. Rev.*, 1946, **53**, 147-158.

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APPENDIX A

Tasks Used in the Measurement of Learning Sets

IVA1: Equivalence of $1x$ and x

Which term is the same as x? $3x$; $10x$; $-2x$; y; $1x$; $8x$; $-y$

IVA2: Identification of an Equation

Which of the following is an equation?

$8y + 2 - 10$	$(8y + 2)10$
$8y + 2 = 10$	$\frac{8y + 2}{10}$

IVA3: Obtaining Products with Zero

Which of the following are equal to zero?

$5g \cdot 3$	$4r \cdot 0$
$2 \cdot 11$	$16 \cdot 8k$
$0 \cdot 157$	$7f \cdot 7e$

IV1: Procedural Order

The order to be followed in solving an equation is:

LCRD	C—Collect like terms
RCLD	D—Divide by the coefficient of "x"
LRCD	R—Transpose all other terms to the right
DRCL	L—Transpose all "x" terms to the left

IV2: Recognition of Equivalent Terms

Which term on the right can be combined with (added to or subtracted from) each term on the left? For every term on the left there is one term on the right which can be combined with it.

1. $3(a - b)$	$14s$	$9q$	$5m$	$-6n$
2. $12n$	$5p$	40	$-4a$	$1pr$
3. $6ab$	$-12ab$	$\frac{11}{x}$	$a - b$	$\frac{2}{y}$
4. $\frac{5}{y}$	$16a$		$(x - y)$	
5. 16				

IV3: Performing Addition and Subtraction of Numbers in Sequence

$$3 + 9 + 27 - 13 + 2 - 39 - 5 + 68 =$$

IV4: Recognizing Equivalence of Multiplication and Division Terms

Which three terms mean $34a$ *multiplied by* bc?

$34a \cdot bc$	$34a + bc$	$\frac{34a}{bc}$
$34a - bc$	$34ab \cdot c$	$34abc$
$34 \cdot \frac{a}{bc}$	$34a = bc$	$34a \div bc$

IV5: Performing Multiplication of Numbers in Sequence

Multiply: 2, 8, 6

IV6: Division of Parenthetical Terms

$$\frac{7(2a + b - 5)}{3b(2a + b - 5)} =$$

IIIA1: Combining Fractions with Like Denominators

$$\frac{3}{2x + y} + \frac{4}{2x + y} - \frac{2}{2x + y} =$$

IIIA2: Simplifying Fractional Expressions

$$\frac{3 \cdot 8 \cdot 4 \cdot 9 \cdot 60}{10 \cdot 3 \cdot 12 \cdot 2} =$$

III1: Identifying Needed Operations in Order

Given this equation and the steps in its solution (in mixed-up order), place the steps in their proper order and label the operation to be done on each step (in order to get to the next step). Use the steps listed in Question 4.

Equation: $4x - 2 = x + 3$ Steps: $3x = 5$

$4x - x - 2 = 3$

$4x - x = 3 + 2$

$x = \frac{5}{3}$

III2: Addition and Subtraction of Terms in Sequence

$2y + 7z - 8y - 4w + y + 14w - z =$

III3: Supplying Sum and Difference Equivalents to Sums and Differences (Arithmetic Numbers)

$2 + 7 - 5 + 1 - 6 = 3 - 6 + 8 + 2 + ?$

III4: Supplying Product and Quotient Equivalents to Products and Quotients (Arithmetic Numbers)

$\frac{2 \cdot 4 \cdot 3}{6 \cdot 2} = \frac{2 \cdot 3 \cdot 4}{3 \cdot 6} \cdot ?$

II1: Supplying Sum and Difference Equivalents to Sums and Differences (Terms)

$3g + 5f + h = 2g - 3f - 4h + ?$

II2: Simplifying an Equation by Adding and Subtracting Arithmetic Numbers to Both Sides

Solve for w: $16 + w - 10 = 7 + 4 - 2$

II3: Simplifying an Equation by Multiplying and Dividing Both Sides by Arithmetic Numbers

Solve for t: $\frac{5t}{4} = 10$

II4: Supplying Product and Quotient Equivalents to Products and Quotients (Terms)

$\frac{3s \cdot 5r \cdot q}{10} = \frac{s \cdot 4q}{10r} \cdot ?$

I1: Simplifying an Equation by Adding and Subtracting Terms to Both Sides

Solve for b: $7b + 2a + 3b - a = 10b + 2a + 35 - b$

I2: Simplifying an Equation by Multiplying, Dividing, Adding, and Subtracting Arithmetic Numbers

Solve for a: $20 = 18 + \frac{9a}{3} - 1$

I3: Simplifying an Equation by Multiplying and Dividing Both Sides by Terms

Solve for e: $\frac{6(3 - e)}{3s(3 - e)e} = 1$

APPENDIX B

Test for Equation Solving

1. Solve for b: $2b - 3 - 8b - 4 + 3b = 13 - 6 - 3b - 2 - 6b$
2. Solve for z: $\frac{3z + 4}{z} = \frac{4 \cdot 6 \cdot 2}{8}$
3. Solve for a: $\frac{3a \cdot 5 \cdot 2}{10} - 12 = \frac{10 \cdot 9 \cdot 4 \cdot 2}{2 \cdot 5 \cdot 4} - 2a$
4. Solve for y: $\frac{4y + 12}{3} = 12$
5. Solve for m: $\frac{6m - 3m}{4} = m - 2$
6. Solve for x: $\frac{4x}{2} = 6x - 8$
7. Solve for a: $\frac{4a}{2} + \frac{6a}{2} + b = 2a + 4$
8. Solve for p: $\frac{2p + 30}{p} = 14 - 4 + 2$
9. Solve for x: $7x + 4x = 3a + 3a + 2a - x$
10. Solve for g: $2g - 3h + 2 + g = 7 - 8h - 2g$

Transfer Test (Equations)

1. Solve for F: $\frac{(F + 4)}{2} + \frac{(3F - 8)}{2} = 5F - 2$
2. Solve for L: $\frac{4 + L + (4 + 3L) + 6 + (4L \cdot 0)}{3} = \frac{3L \cdot 4 \cdot 2 \cdot 14}{8 \cdot 7}$
3. Solve for p: $-2 + 3p + 3q - r - p + q + 3p + 8 - 2r = 18 + 4q - 6 + 3p - r$
4. Solve for i: $i - \frac{12i}{4} + \frac{4i}{3} - \frac{i}{3} + \frac{4i}{3} = 10$
5. Solve for g: $4g + \frac{3g - 6 - g}{} = 12$
6. Solve for N: $\frac{3N + N - 2}{N + 2N - 4} = 10 + 4 - 12$
7. Solve for c: $\frac{6a - 6b}{3c(a - b)} + \frac{3}{c} - 2 = 5 - 2$
8. Solve for T: $4Tx - 4Ty = (5x + 3y)(x - y)$
9. Solve for Q: $\frac{3Q(6Q - 4)}{Q} = \frac{300}{2} + \frac{3}{2} - \frac{27}{Q} + 3Q$
10. Solve for x: $6x + 4y + z - 2x + y - 4z = \frac{4y}{y - z} - \frac{2z}{y - z} - \frac{3y}{y - z} + \frac{z}{y - z}$

APPENDIX C

TABLE C1

Coefficients of Correlation between Basic Abilities and Rates of Attaining Learning Sets (Product-Moment) and between the Same Abilities and Achievement of Learning Sets (Point-Biserial)

($N = 118$)

Learning set	Ability factors									
	Learning rate					Achievement				
	Number	Symbol Recognition	Integration I	Vocabulary	Speed of Symbol Discrimination	Number	Symbol Recognition	Integration I	Vocabulary	Speed of Symbol Discrimination
IVA1	38	80	60	26	40					
IVA2	40	75	54	24	28					
IVA3	71	76	51	28	33					
IV1	34	48	76	30	28	40	20	53	02	28
IV2	38	74	57	26	18	30	33	30	02	47
IV3	82	54	49	28	28	24	29	05	01	11
IV4	69	71	45	26	33	29	22	11	08	34
IV5	85	53	40	32	40	31	25	14	09	21
IV6	69	69	38	24	28	26	23	20	06	20
IIIA1	64	58	40	22	40	26	52	34	22	33
IIIA2	74	55	34	20	33	40	50	26	14	29
III1						50	49	50	08	42
III2	44	54	34	18	30	59	51	41	24	37
III3	60	55	25	21	24	41	35	27	09	43
III4	62	34	28	24	26	68	60	43	15	47
II1	41	33	41	20	18	62	52	33	17	11
II2	54	42	30	18	33	47	42	44	25	49
II3	53	44	27	24	18	69	59	50	15	57
II4						70	63	34	09	23
I1	38	27	20	19	26	63	57	41	11	36
I2	51	40	43	23	32	70	71	56	20	53
I3	57	22	18	23	40	40	30	[illegible]	10	23

TABLE C2

COEFFICIENTS OF CORRELATION (POINT-BISERIAL) BETWEEN ACHIEVEMENT OF EACH LEARNING SET AND LEARNING RATE OF EACH HIGHER-LEVEL LEARNING SET
(N = 118)

Achievement of learning set	Rate of learning										
	IIIA1	IIIA2	III2	III3	III4	II1	II2	II3	I1	I2	I3
IV1	20	28	31	34	25	26	24	32	28	34	30
IV2	38	18	44	24	32	48	30	25	54	28	29
IV3	39	18	43	46	19	49	50	24	58	44	26
IV4	18	34	22	20	44	19	26	48	28	50	53
IV5	19	33	21	19	45	18	24	49	28	55	64
IV6	21	35	19	17	46	16	26	48	25	52	66
IIIA1			32	24	18	42	24	22	50	18	25
IIIA2			24	20	35	21	18	41	26	52	57
III1						22	20	18	28	26	28
III2						38	26	18	55	24	26
III3						38	46	18	55	44	26
III4						18	20	41	26	44	57
II1									55	34	26
II2									53	54	25
II3									24	48	50
II4									26	34	57